Photocurrent signal to noise ratio

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1 Shot noise limit

The measurability of a photodiode signal is limited by the shot noise of the photocurrent. Shot noise is the statistical uncertainty in the average current due to the quantization of electron charge. Shot noise typically manifests only in semiconductor junctions where each electron crosses a space-charge barrier in an independent fashion. In a metallic resistor, the electrons repel each other, effectively spacing themselves into a smooth stream, and the shot effect vanishes[1]. However, in a photodiode, with a given photocurrent i_d, the rms shot noise contribution is $\sqrt{2qi_d\Delta f}$. The log_2 of the ratio of photocurrent to shot noise puts an upper bound on the the maximum resolvable bits in the system.

i_d	shot noise in 100kHz bw	SNR	bits
1uA	178pA	5607	12
100nA	56pA	1773	10.7
10nA	18pA	560	9

In addition to shot noise from the photo current, there is the possible contribution of noise by the LED itself. It has been shown that driving an LED in constant voltage produces standard shot noise based on the LED drive current[2]. However, driving an LED in constant current mode will reduce the light noise below shot noise level. If the LED is shunted by a capacitor, then at frequencies where the capacitive reactance is lower than the LED impedance, the light noise will contain shot noise. In the analysis that follows, we assume that the LED is driven in a current source regime, and that the drive current is much higher than the received photocurrent, rendering any shot contribution from the LED negligable.

[1] Marc de Jong, Sub-Poissonian shot noise, Physics World, August 1996, page 22

[2] Endo, et al. “Dependence of an LED Noise on Current Source Impedance”, Journal of the Physical Society of Japan, Volume 66, Issue 7, pp. 1986 (1997)

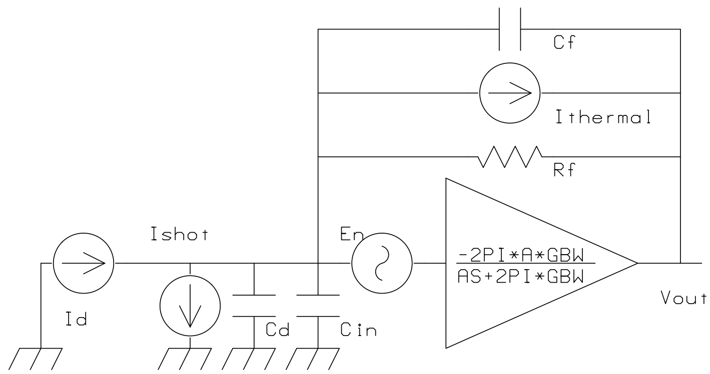

Figure 1: Transimpedance schematic

2 Transimpedance Amplifier Noise Analysis

Figure 1 shows a transimpedance circuit with noise sources and op amp model. In a transimpedance configuration, the dominant noise sources are the opamp input voltage noise e_n plus the shot noise from the photodiode $i_{shot} = \sqrt{2qi_d\Delta f}$ and the thermal noise $i_{thermal} = \sqrt{\frac{4kT\Delta f}{R_f}}$ of the feedback resistor. The input current noise contribution of most opamps is negligable in this configuration and is ignored.

Two gain terms are computed for the circuit. The first is the transimpedance X(s). We define the opamp open-loop gain to be a single pole integrator with a Gain Bandwidth Product G and Open Loop Gain A_0.

$$A(s) = \frac{2\pi A_o G}{A_0 s + 2\pi G} \tag{1}$$

The impedance of the input network from the transimpedance node to ground is

$$ZA = \frac{1}{(C_d + C_{in})\, s} \tag{2}$$

The impedance of the feedback network from output to input is

$$ZB = \frac{R_f}{1 + R_f C_f s} \tag{3}$$

From these two terms we can compute a transimpedance value from input to output

$$X(s) = \frac{-A(s) * ZA * ZB}{ZA + ZB + A(s) * ZA} \tag{4}$$

and a gain for the opamp noise voltage to the output

$$E(s) = \frac{-A(s) * (ZA + ZB)}{ZA + ZB + A(s) * ZA} \tag{5}$$

The output noise is computed by combining the frequency-weighted noise sources in rms fashion

$$V_{out}(s) = \sqrt{X(s) i_{shot}^2 + X(s) i_{thermal}^2 + E(s) i_e^2} \tag{6}$$

This noise spectrum is integrated from DC to the unity gain frequency of X(s) to compute the expected sigma of the output signal

$$\sigma_{xamp} = \sqrt{\int_0^\infty [V_{out}(s)]^2 \, ds} \tag{7}$$

Finally the system signal to noise ratio is calculated by multiplying the input current by the DC transimpedance and dividing by the output noise

$$SNR = \frac{i_d * R_f}{\sigma_{xamp}} \tag{8}$$

3 Switched Integrator Noise Analysis

Figure 2 shows the switched integrator with noise sources and op amp model. The circuit differs from the transimpedance circuit by the absence of the feedback resistor and associated thermal noise and the addition of reset noise in the hold capacitor.

Because the feedback resistor is missing, we compute $E(s)$ and $X(s)$with a redefined feedback impedance

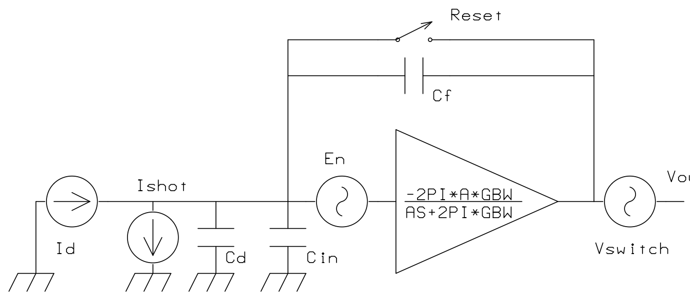

Figure 2: Switched integrator schematic

$$ZB = \frac{1}{C_f s} \tag{9}$$

There is an extra noise associated with the operation of the switch. Whenever a capacitor is connected to a voltage source, there is an average charge Q=CV and a random variation of charge due to thermal motion of the electrons. When the capacitor is disconnected, the random fluctuation is "frozen" and a random voltage variation $v_{switch}$is sampled across the capacitor.

$$V_{switch} = \sqrt{K * T/C_f} \tag{10}$$

The output noise is computed by combining the frequency-weighted noise sources in rms fashion

$$V_{out}(s) = \sqrt{X(s)i_{shot}^2 + E(s)i_e^2} \tag{11}$$

The voltage on the feedback capacitor is bounded by periodically zeroing the the charge by shorting it with a reset switch. This can be modelled by subtracting the value of the output signal at time t from the value at $t+t_s$. This is equivalent to convolving the signal with a delta pulse minus a delayed delta pulse, which in the Laplace domain is equivalent to multiplying by

$$W(s) = 1 - e^{-st_s} \tag{12}$$

The weighted ouput noise spectrum is integrated to compute the expected sigma of the output signal

$$\sigma_{int} = \sqrt{V_{switch}^2 + \int_0^\infty [W(s)V_{out}(s)]^2 \, ds} \tag{13}$$

giving a final SNR

$$SNR = \frac{[i_d t_s / C_f]}{\sigma_{int}} \tag{14}$$

If the switch noise and op-amp voltage noise are taken to be zero, this simplifies to the ideal shot-limited SNR

$$SNR_{ideal} = \sqrt{\frac{i_d t_s}{2q}} \tag{15}$$

The integrator, at high currents, has an SNR that improves as the square root of both the photocurrent and the integration time. The feeback capacitance only affects the switch and op-amp noise contributions.

4 Performance Comparison with Typical Components

For comparative analysis, we use an AD8608 opamp as the signal amplifier for both topologies. The input voltage noise e_n is typically 8nV per root Hz, and the input current i_n is 0.01 pA per root Hz. The Gain Bandwidth Product is 10e6 and the Open Loop Gain is 1e6. The opamp input capacitance is 2pF. The nominal circuit values are Rf=300k, Cf=5p for the transimpedance amp, and Cf=10p, ts = 40uS for the switched integrator. SNR values are given for both approaches at three input currents.

Numerical Integration was done with a custom C++ program, available upon request.

	xamp (Rf=300k, cf=5p)		switched integrator (10us, 10p)	
	70p	11p	70p	11p
SNR at 1uA	2089	3161	3933	4346
SNR at 100nA	234	434	678	1003
SNR at 10nA	24	45	76	133

Both topologies can be improved. The xamp design benefits from increasing the feedback resistor to 600k while reducing Cf to 2.5pF.

	xamp (Rf=600k, Cf=2.5p)	
	70p	11p
SNR at 1uA	2691	3688
SNR at 100nA	331	613
SNR at 10nA	34	68

The integrator design can be improved by increasing the integration time. The effective noise bandwidth of an integrate and dump circuit is 1/ts. Changing the feedback capacitance value has a small effect.

	switched integrator (40us, 10p)	
	70p	11p
SNR at 1uA	8770	9034
SNR at 100nA	2109	2588
SNR at 10nA	291	490

5 Discussion and Improvements

The optimized integrator has about 9 times the SNR of the best transimpedance design at 10nA power levels. If a three-sigma criteria is required, a transimpedance amp design would be capable of only a 10% touch detection down to 10nA photocurrent. The switched integrator, by comparison, would be able to resolve a 1% drop in power within a single measurement.

The transimpedance system can be further improved, however. The amplifier has an effective bandwidth of 100kHz. The impulse response of such a system has a time constant of $\frac{1}{2\pi 100e3} = 1.6us$, and a 10/90 risetime of $0.35/BW$, or 3.5 us. Making a transimpedance measurement involves waiting for a time long enough to purge the previous photocurrent signal and then sampling the output one or more times. If we wish to purge the system to less than 1% we need to wait five time constants, $\frac{1}{e^5} = .006$. This is a dead time of 1.6*5=8us. To make the system comparable to the integrating amp, we can use up to 40us total to make a complete measurement. This gives us a 32us window for multiple samples. Multiple averaged samples can be modeled by convolving with an appropriate weighting function prior to integration of the noise spectrum.

If the total integration window is t_s, and the signal is sampled by the ADC at n equally spaced points within the window, the appropriate weighting function in the Laplace domain is

$$W(s) = \frac{1}{n}\sum_{k=0}^{n-1} e^{-\frac{kst_s}{n}} = \frac{1}{n}\sum_{k=0}^{n-1}\left\{cos(\frac{-kst_s}{n}) + i * sin(\frac{-kst_s}{n})\right\} \tag{16}$$

The formula for xamp noise is then modified to include the weighting factor

$$\sigma_{xamp} = \sqrt{\int_0^\infty [W(s)V_{out}(s)]^2\, ds} \tag{17}$$

6 Conclusions

Figure 3 shows the oversampled transimpedance amp SNR with both 11 and 70pF diode capacitance vs the switched integrator with n as a parameter. The measurement window allows for 8uS settling to clear out any effect of the previous measurement, and then 32uS of total measurement time, broken up into n ADC readings which are averaged together. The xamp starts from about a 10x performance penalty and asymptotically approaches 90% of the switched integrator performance with 100 samples. For modest numbers of samples, say 4, the transimpedance amp has about 2.5x less SNR than the switched integrator circuit. The difference between the two circuits is the added Johnson noise of the feedback resistor, which cannot be completely overcome with averaging.

The xamp is unable to achieve 1% touch detect with three sigma reliability (eg: SNR=300) even with 10x oversampling and averaging. The averaged xamp improves substantially in performance but is still limited by the thermal noise from the feedback resistor. An integrator with 40us averaging is capable of 1% touch detection with 3 sigma reliability with no further processing required.

A 1% touch detect performance with a transimpedance amp can be achieved with higher optical power at the cost of reduced battery life in portable equipment. The transimpedance amp may be a viable option for simplified prototype construction with discrete components. For a custom IC designed for portable equipment, it may be best to use a switched integrator preamp to save system power.

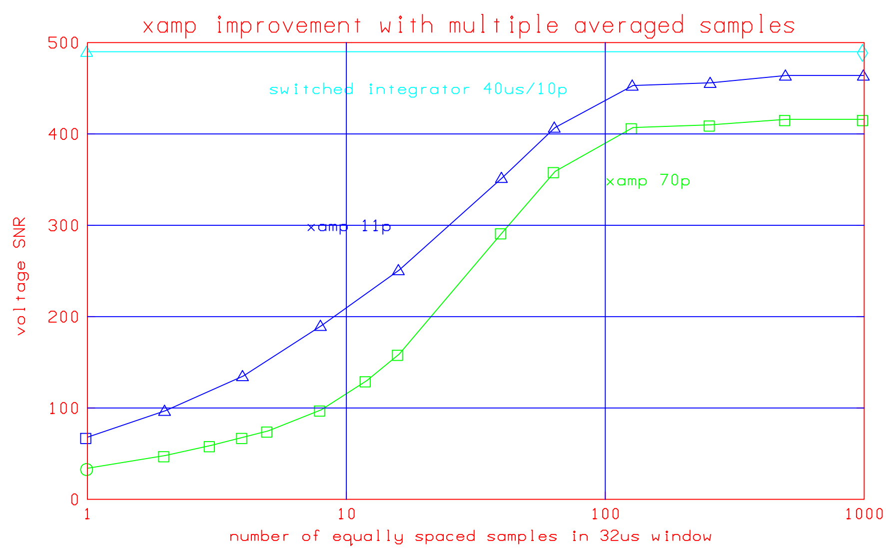

Figure 3: Comparison of transimpedance amp with both 11 and 70pF with switched integrator performance with number of averaged ADC readings within a 32uS window as a parameter.